Number System

1) Find which of the variables x, y, z and u represent rational numbers and which irrational numbers:

(i) $x^2 = 5$ (ii) $y^2 = 9$ (iii) $z^2 = .04$ (iv) $u^2 = \frac{17}{4}$

2) Find three rational numbers between

(i) –1 and –2 **(ii) 0.1 and 0.11**

(iii) $\frac{5}{7}$ and $\frac{6}{7}$ (iv) $\frac{1}{4}$ and $\frac{1}{5}$

3) Insert a rational number and an irrational number between the following :

(i) 2 and 3 (ii) 0 and 0.1 (iii) $\frac{1}{3}$ and $\frac{1}{2}$

(iv) $\frac{-2}{5}$ and $\frac{1}{2}$ (v) 0.15 and 0.16 (vi) $\sqrt{2}$ and $\sqrt{3}$

(vii) 2.357 and 3.121 (viii) .0001 and .001 (ix) 3.623623 and 0.484848

(x) 6.375289 and 6.375738

4) Represent the following numbers on the number line :

$7, 7.2, \frac{-3}{2}, \frac{-12}{5}$

5) Locate $\sqrt{5}$, $\sqrt{10}$ and $\sqrt{17}$ on the number line.

6) Represent geometrically the following numbers on the number line :

(i) $\sqrt{4.5}$ (ii) $\sqrt{5.6}$ (iii) $\sqrt{8.1}$ (iv) $\sqrt{2.3}$

7) Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

(i) 0.2 (ii) 0.888... (iii) $5.\overline{2}$ (iv) $0.\overline{001}$

(v) 0.2555... (vi) $0.1\overline{34}$ (vii) .00323232... (viii) .404040...

8) Show that $0.142857142857... = \frac{1}{7}$

9) Simplify the following:

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

(ii) $\dfrac{\sqrt{24}}{8}+\dfrac{\sqrt{54}}{9}$

(iii) $\sqrt[4]{12}\times\sqrt[7]{6}$

(iv) $4\sqrt{28}\div 3\sqrt{7}\div\sqrt[3]{7}$

(v) $3\sqrt{3}+2\sqrt{27}+\dfrac{7}{\sqrt{3}}$

(vi) $\left(\sqrt{3}-\sqrt{2}\right)^2$

(vii) $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$

(viii) $\dfrac{3}{\sqrt{8}}+\dfrac{1}{\sqrt{2}}$

(ix) $\dfrac{2\sqrt{3}}{3}-\dfrac{\sqrt{3}}{6}$

10) Rationalise the denominator of the following:

(i) $\dfrac{2}{3\sqrt{3}}$

(ii) $\dfrac{\sqrt{40}}{\sqrt{3}}$

(iii) $\dfrac{3+\sqrt{2}}{4\sqrt{2}}$

(iv) $\dfrac{16}{\sqrt{41}-5}$

(v) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$

(vi) $\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

(vii) $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

(viii) $\dfrac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

(ix) $\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

11) Find the values of a and b in each of the following:

(i) $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}}=a-6\sqrt{3}$

(ii) $\dfrac{3-\sqrt{5}}{3+2\sqrt{5}}=a\sqrt{5}-\dfrac{19}{11}$

(iii) $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=2-b\sqrt{6}$

(iv) $\dfrac{7+\sqrt{5}}{7-\sqrt{5}}-\dfrac{7-\sqrt{5}}{7+\sqrt{5}}=a+\dfrac{7}{11}\sqrt{5}b$

12) If $a = 2+\sqrt{3}$, then find the value of $a - \frac{1}{a}$.

13) Rationalise the denominator in each of the following and hence evaluate by taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, upto three places of decimal.

(i) $\frac{4}{\sqrt{3}}$ (ii) $\frac{6}{\sqrt{6}}$ (iii) $\frac{\sqrt{10}-\sqrt{5}}{2}$

(iv) $\frac{\sqrt{2}}{2+\sqrt{2}}$ (v) $\frac{1}{\sqrt{3}+\sqrt{2}}$

14) Simplify :

(i) $\left(1^3+2^3+3^3\right)^{\frac{1}{2}}$ (ii) $\left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6$

(iii) $\left(\frac{1}{27}\right)^{\frac{-2}{3}}$ (iv) $(625)^{-\frac{1}{2}\left(-\frac{1}{4}\right)^2}$

(v) $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$ (vi) $64^{-\frac{1}{3}}\left(64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right)$

(vii) $\frac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$

15) Let x and y be rational and irrational numbers, respectively. Is $x+y$ necessarily an irrational number? Give an example in support of your answer.

16) Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

17) State whether the following statements are true or false? Justify your answer.

(i) $\frac{\sqrt{2}}{3}$ is a rational number.

(ii) There are infinitely many integers between any two integers.

(iii) Number of rational numbers between 15 and 18 is finite.

(iv) There are numbers which cannot be written in the form $\frac{p}{q}$, $q \neq 0$, p, q both are integers.

(v) The square of an irrational number is always rational.

(vi) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.

(vii) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}$, $q \neq 0$ and so it is a rational number.

18) Classify the following numbers as rational or irrational with justification :

(i) $\sqrt{196}$ (ii) $3\sqrt{18}$ (iii) $\sqrt{\frac{9}{27}}$ (iv) $\frac{\sqrt{28}}{\sqrt{343}}$

(v) $-\sqrt{0.4}$ (vi) $\frac{\sqrt{12}}{\sqrt{75}}$ (vii) 0.5918

(viii) $\left(1+\sqrt{5}\right)-\left(4+\sqrt{5}\right)$ (ix) 10.124124... (x) 1.010010001...

19) Express $0.6+0.\overline{7}+0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

20) Simplify : $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$.

21) If $\sqrt{2}=1.414$, $\sqrt{3}=1.732$, then find the value of $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$.

22) If $a=\frac{3+\sqrt{5}}{2}$, then find the value of $a^2+\frac{1}{a^2}$.

23) If $x=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$.

24) Simplify : $(256)^{-\left(4^{\frac{-3}{2}}\right)}$

25) Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$